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Derivation of pair correlation function from an equation of weakly correlated plasma

Anirban Bose*

Abstract

Plasma can be thought of as a many body system where coulomb interaction holds the key to determine its statistical nature . Lowering the temperature and raising the density of plasma gradually shifts the balance in a continuous manner so that the individual effects in the form of binary collision becomes important and drifts the system to the so called 'correlated plasma systems'. This is the regime where discrete nature of plasma begins to take effect. Consequently detail kinetic theory is needed to explore the system under consideration. In this context, an equation of pair correlation function has been derived from the Bogoliubov-Born-Green-Kirkwood-Yvon (BBGKY) hierarchy for inhomogeneous dusty plasma under certain approximations. A solution of this equation has been obtained under certain conditions.

Keywords:

First keyword; Statistical Mechanics
Second keyword; Plasma Physics
Third keyword; BBGKY hierarchy

Author correspondence:

Anirban Bose,
Department of Physics
Serampore College Serampore, Hooghly, India
Email: sercolanirban@gmail.com

1. Introduction

The word PLASMA originates from Greek, meaning "something formed or molded" and was taken into light by Tonk and Langmuir in 1929 to study the characteristics of ionized gases [1]. It is termed as the fourth state of matter and claimed to build up 99.9 percent of this universe. However, the natural climate of earth does not make it as plentiful as expected with respect to its abundance in the universe. Therefore, plasmas are cooked up in laboratories by passing electric current or shining radio waves in gaseous mediums. Solar environment is particularly favorable for their existence. It also remains in the interstellar space. Stars like Sun produce stream of highly energetic charged particles(plasma), mainly constituted by protons and electrons, are called "solar wind" [2]and their interaction with the earth magne-tosphere acts like an electric field generator[magnetospheric magnetoHydroDynamic(MHD) generator][3], giving rise to electric field in the magnetopause. Collision between the charged particles and neutral atoms in the atmosphere leads to colourful emissions termed as "au-roras". Plasma has its existence in the ionosphere. Over the year a variety of Plasma based experimental device like Fusion machines (RFP, Tokamaks, Stellarators), Space plasma simulation chambers (Q-machines, Mirror Machines)[4,5], Gas discharges have been developed to facilitate its application in different directions. This field is intimately connected to other branches of science like astrophysics, medical science, molecular physics, atmospheric physics and put up a major contribution in their upliftment.

Plasma can be thought of as a many body system where coulomb interaction holds the key to determine its statistical nature. The simplest example of that kind is the system of a set of noninteracting particles. Plasma, in a way, is compared with the ideal gas. Even though plasma particles are always involved in electrostatic

* Serampore College, Serampore, Hooghly, India

interaction among themselves the strength of the average interaction energy is well under so that the plasma thermodynamic functions can be obtained by the virial expansion about the thermodynamic function of an ideal gas. Under this circumstance plasma is called "ideal plasma". "Correlation", the tendency of like charges to stay apart and the trend of group formation of unlike charges, shifts plasma away from its ideal nature.

Neglecting correlation does not extinguish the entire interaction of the system. Interaction on a single plasma particle can be broadly classified in two categories: contribution from the far away particles where the density instead of the exact locations of the particles get the primary attention and the remaining portion is provided by the nearest neighbors (through binary collision). The former part works like an average effect and build up the "collective" nature of the system. On the contrary, the nearest neighbors constitute the "individual" counterpart (single particle effect). Far away particles working like groups masks the individual participation coming in the form of nearest neighbor interactions as long as low density and high temperature prevails in the medium, signifying the dominance of collective effect over the discrete nature and a major part of plasma research is concerned in that limit. There are two common techniques to deal with ideal plasma systems. First of them is the fluid theory where the plasma is treated as a fluid element. If the particles in a small volume have the similar reaction to the perturbation they can execute coherent motion like an element of normal fluid [6]. A great deal of study of plasma collective phenomenons (waves, quasi-steady states, instabilities) is carried out successfully with the approach. This approach can work as long as the thermal motions of the particles are insignificant. The other approach , popularly known as "vlasov theory" overcomes this shortcoming of the previous technique and gives a broader microscopic view of the system. This is all about ideal plasma.

Lowering the temperature and raising the density gradually shifts the balance in a continuous manner so that the individual effects in the form of binary collision begins to glow and drifts the system to the so called 'correlated plasma systems', another hot topic in plasma community. This is the regime where discrete nature of plasma begins to take effect.consequently detail kinetic theory is needed to explore the system under consideration.

Plasma characteristic drifts gradually with its coupling parameter. Its higher value indicates the importance of single particle effect over the collective phenomenon. So small but finite value of Coupling parameter forces to take on the microscopic route, starting from the N-particle Liouville equation, eventually leading to a set of N coupled integro-diferential equation. The series is broken by BBGKY hierarchy and restricted to a close set consisting of a couple of equations containing the single and two particle distribution functions.

The problem of solving the BBGKY hierarchy of equations is as hard as solving the original Liouville equation. However, the equations are in such a form that some kind of closure relation may be employed (by the implementation of various approximations according to the nature of the problem at hand) for the meaningful truncation of the chain [8]. In this Communication, an equation of pair correlation function has been found from the first and second members of BBGKY hierarchy in a weakly correlated inhomogeneous plasma system under certain approximations. This equation has been solved under certain approximations. Therefore, this study may play an important role to study the homogeneous and inhomogeneous many body charged particle systems.

In a dusty plasma [7] composed of electrons, ions and dust particles, the coupling constant of the dust component can attain a wider range of values compared to ions or electrons due to the presence of high charges on the dust particles. In this work we are interested in a dusty plasma system where the dusts are weakly correlated with the ions and electrons are uncorrelated.

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2. Research Method

In order to obtain the pair correlation equation, we consider the first two members of the BBGKY hierarchy [8](for the dusts) where the three particle correlation function has been neglected.

$$\frac{\partial f_1}{\partial t}+\vec{v}_1.\frac{\partial f_1}{\partial \vec{x}_1}+n_0\int dX_2\vec{a}_{12}.\frac{\partial}{\partial \vec{v}_1}[f_1(X_1)f_1(X_2)+g_{12}]=0 \quad (1)$$

$$\frac{\partial g_{12}}{\partial t}+\vec{v}_1.\frac{\partial g_{12}}{\partial x_1}+\vec{v}_2.\frac{\partial g_{12}}{\partial x_2}+n_0\int dX_3 f_1(X_3)[\vec{a}_{13}.\frac{\partial g_{12}}{\partial \vec{v}_1}+\vec{a}_{23}.\frac{\partial g_{12}}{\partial \vec{v}_2}]=$$
$$-[\vec{a}_{12}.\frac{\partial}{\partial \vec{v}_1}+\vec{a}_{21}.\frac{\partial}{\partial \vec{v}_2}][f_1(X_1)f_1(X_2)+g_{12}]$$
$$-n_0\int dX_3 f_1(X_3)[\vec{a}_{13}.\frac{\partial f_1(X_1)}{\partial \vec{v}_1}g_{23}+\vec{a}_{23}.\frac{\partial f_1(X_2)}{\partial \vec{v}_2}g_{13}] \quad (2)$$

where f(X)=f (x,v), f_1 is the single particle distribution, g_{12} is the pair correlation function and

$$\vec{a}_{ij}=-\frac{1}{m}\frac{\partial \phi_{ij}}{\partial \vec{x}_i}$$

These terms denote acceleration of the I-th particle due to force exerted by the j-th particle and ϕ_{ij} is the energy of interaction between them.

For a plasma in equilibrium the time derivatives of single particle distribution and pair correlation function do not exist in eqs. (1) and (2), the pair correlation function g_{12} is written according to the following

$$g_{12}(X_1,X_2)=f_1(X_1)f_1(X_2)\chi_{12}(x_1,x_2) \quad (3)$$

and the single particle distribution functions f_1 (X_1) and f_1 (X_2) are functions of both position and velocity. By making use of eqs. (1) and (3) in the equilibrium condition,

$$\vec{v}_2.\frac{\partial g_{12}}{\partial \vec{x}_2}=f_1(X_1)\chi_{12}\vec{v}_2.\frac{\partial f_1(X_2)}{\partial \vec{x}_2}+f_1(X_1)f_1(X_2)\vec{v}_2.\frac{\partial \chi_{12}}{\partial \vec{x}_2} \quad (4)$$

$$\vec{v}_2.\frac{\partial f_1(X_2)}{\partial \vec{x}_2}=-n_0\int dX_3\vec{a}_{23}.\frac{\partial}{\partial \vec{v}_2}[f_1(X_2)f_1(X_3)+g_{23}] \quad (5)$$

and similar expressions for $v_1\cdot\partial g_{12}/\partial x_1$. Eqs. (4) and (5) lead to the following form of eq. (2) under equilibrium conditions :

$$f_1(X_1)f_1(X_2)(\vec{v}_1.\frac{\partial \chi_{12}}{\partial \vec{x}_1}+\vec{v}_2.\frac{\partial \chi_{12}}{\partial \vec{x}_2})=$$
$$-n_0\int dX_3[f_1(X_2)\chi_{12}\vec{a}_{13}.\frac{\partial g_{13}}{\partial \vec{v}_1}+f_1(X_1)\chi_{12}\vec{a}_{23}.\frac{\partial g_{23}}{\partial \vec{v}_2}]$$
$$-[\vec{a}_{12}.\frac{\partial}{\partial \vec{v}_1}+\vec{a}_{21}.\frac{\partial}{\partial \vec{v}_2}][f_1(X_1)f_1(X_2)+g_{12}]$$
$$-n_0\int dX_3 f_1(X_3)[\vec{a}_{13}.\frac{\partial f_1(X_1)}{\partial \vec{v}_1}g_{23}+\vec{a}_{23}.\frac{\partial f_1(X_2)}{\partial \vec{v}_2}g_{13}] \quad (6)$$

It is necessary to know the single particle distribution function as well as the average interaction potential between two particles. Since the latter is known to be of the Debye-Hückel type for a plasma in thermal equilibrium, we choose the corresponding type of single particle distribution function with Maxwellian velocity distribution. Thus the single particle distribution functions are written in the following form:

$$f_1(X_1)=f_M(v_1)F_1(X_1)$$

where f_M is a Maxwellian distribution and F_1 denotes the spatially dependent part of the single particle distribution function.It indicates the absence of velocity-space correlation. But for relativistic plasma the current constituted by moving charges can not be overlooked and that produces magnetic field and a velocity dependent term takes place in the interaction potential, giving rise to velocity-space correlation.

Inserting g_{23} and g_{13}

$$f_1(X_1)f_1(X_2)(\vec{v}_1.\frac{\partial \chi_{12}}{\partial \vec{x}_1}+\vec{v}_2.\frac{\partial \chi_{12}}{\partial \vec{x}_2})=$$
$$-\frac{1}{k_BT}(\vec{v}_1.\frac{\partial \phi_{12}}{\partial \vec{x}_1}+\vec{v}_2.\frac{\partial \phi_{12}}{\partial \vec{x}_2})f_1(X_1)f_1(X_2)(1+\chi_{12})$$
$$f_1(X_1)f_1(X_2)\frac{n_0 m}{k_BT}\int dX_3[\vec{a}_{13}.\vec{v}_1 f_1(X_3)\chi_{23}$$

$$\vec{a}_{23}.\vec{v}_2 f_1(X_3)\chi_{13}] - f_1(X_1)f_1(X_2)\frac{n_0 m}{k_B T}\int dX_3$$

$$[\vec{a}_{13}.\vec{v}_1 f_1(X_3)\chi_{13}\chi_{12} + \vec{a}_{23}.\vec{v}_2 f_1(X_3)\chi_{23}\chi_{12}] \quad (7)$$

We can write,

$$f_1(X_1)f_1(X_2)\vec{A}.\vec{v}_1 + f_1(X_1)f_1(X_2)\vec{B}.\vec{v}_2 = 0 \quad (8)$$

where

$$A = \frac{\partial\chi_{12}}{\partial\vec{x}_1} + \frac{1}{k_B T}\frac{\partial\phi_{12}}{\partial\vec{x}_1}(1+\chi_{12}) + \frac{n_0}{k_B T}\int dX_3 \frac{\partial\phi_{13}}{\partial\vec{x}_1} f_1(X_3)\chi_{23}$$

$$-\frac{n_0\chi_{12}}{k_B T}\int dX_3 \frac{\partial\phi_{13}}{\partial\vec{x}_1} f_1(X_3)\chi_{13} \quad (9)$$

$$B = \frac{\partial\chi_{12}}{\partial\vec{x}_2} + \frac{1}{k_B T}\frac{\partial\phi_{12}}{\partial\vec{x}_2}(1+\chi_{12}) + \frac{n_0}{k_B T}\int dX_3 \frac{\partial\phi_{23}}{\partial\vec{x}_2} f_1(X_3)\chi_{13}$$

$$-\frac{n_0\chi_{12}}{k_B T}\int dX_3 \frac{\partial\phi_{23}}{\partial\vec{x}_2} f_1(X_3)\chi_{23} \quad (10)$$

For arbitrary and linearly independent v_1 and v_2, A and B both vanish. Therefore,

$$\frac{\partial\chi_{12}}{\partial\vec{x}_1} + \frac{1}{k_B T}\frac{\partial\phi_{12}}{\partial\vec{x}_1}(1+\chi_{12}) + \frac{n_0}{k_B T}\int dX_3 \frac{\partial\phi_{13}}{\partial\vec{x}_1} f_1(X_3)\chi_{23} - \frac{n_0\chi_{12}}{k_B T}\int dX_3 \frac{\partial\phi_{13}}{\partial\vec{x}_1} f_1(X_3)\chi_{13} = 0 \quad (11)$$

This is the equation of the pair correlation function derived from the BBGKY hierarchy.

3. Results and Analysis

It can be further simplied in the limit χ <<1. We can drop the third and last term of the equation to obtain

$$\frac{\partial\chi_{12}}{\partial\vec{x}_1} + \frac{1}{k_B T}\frac{\partial\phi_{12}}{\partial\vec{x}_1} + \frac{n_0}{k_B T}\int dX_3 \frac{\partial\phi_{13}}{\partial\vec{x}_1} f_1(X_3)\chi_{23} = 0 \quad (12)$$

Hence,

$$\chi_{12} = -\frac{\phi_{12}}{k_B T} - \frac{n_0}{k_B T}\int dX_3\phi_{13} f_1(X_3)\chi_{23} \quad (13)$$

Performing the velocity integral

$$\chi_{12} = -\frac{\phi_{12}}{k_B T} - \frac{1}{k_B T}\int dr_3\phi_{13} n_1(r_3)\chi_{23} \quad (14)$$

The pair correlation equation is obtained in the weak coupling limit.
It can be further simplified In the limit $r_{12} << \lambda_D$

$$\chi_{12} = -\frac{\phi_{12}}{k_B T} \quad (15)$$

For, homogeneous system the pair correlation function is obtained as

$$\chi_{12} = \frac{1}{k_B T}\frac{\exp(-k_B r_{12})}{r_{12}} \qquad (16)$$

Therefore, pair correlation function has been derived for different physical situations in a weakly coupled plasma system.

4. Conclusion

In this Communication, we have obtained an equation of the pair correlation function and solved it under the equilibrium condition of a dusty plasma with the weakly correlated dusts and uncorrelated ions and electrons. This equation is derived by solving the first two equations of BBGKY hierarchy in the absence of velocity-space correlation. In this process, the three particle correlation has been neglected under the weakly correlated condition. In the limit $\chi_{12} << 1$, Eq. (14) looks similar to the Ornstein-Zernike (OZ) equation [9]. But it cannot be called the OZ equation anyway. The original OZ equation relates the two unknown functions: total and direct correlations [10-13] (for inhomogeneous systems, it also includes the one-particle density function instead of the simple density of homogeneous systems), and it is unclosed. The equation derived in this article contains a given interaction potential / (i.e., some known function) as well as one unknown function v, which is related to the total correlation (for inhomogeneous systems, the equation also includes the one-particle density function). It is closed and hence needs no closure relation[14] because it is based on the truncated (and closed) set of BBGKY Equations (1) and (2) from Ref. 1 (where the three-particle correlation is neglected) instead of the actual unclosed BBGKY hierarchy. The OZ equation and Eq. (14) may seem similar, but, in fact, there is no real similarity between them.

This equation has been solved for the homogeneous system in the limit $\chi <<1$. The result seems to be consistent with the previous investigations performed in this field [15]. For the inhomogeneous case the pair correlation function has been obtained in the limit r_{12}(separation between particle 1 and particle 2 $<< \lambda_D$(Debye length). This solution can be inserted in the first member of BBGKY hierarchy and it can be solved to obtain the single particle distribution function with a modified effective potential which takes care of the two particle coorelation in the inhomogeneous system.

However, the scope of application of Eq. (11) should be properly explored. For this purpose, we can tally the results obtained from this equation with the experimental results. Moreover, we emphasize that Eq. (11) is valid as long as we are permitted to ignore the three particle correlation, and only in the limit $\chi_{12} << 1$, it further reduces to Eq. (14) which appears to be formally similar to the OZ equation. The successful completion of this investigation will eventually establish the reliability of the equation and its further applications in case of inhomogeneous charged particle systems.

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